

Hong Kong Mathematics Olympiad (2007 – 2008)

Heat Event (Group)

香港数学竞赛 (2007 – 2008)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知 $5 + \sqrt{11}$ 的小数部分为 A 及 $5 - \sqrt{11}$ 的小数部分为 B 。设 $C = A + B$ ，求 C 的值。

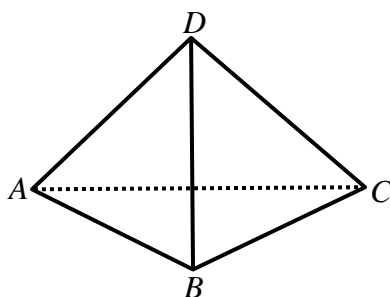
Given that the decimal part of $5 + \sqrt{11}$ is A and the decimal part of $5 - \sqrt{11}$ is B . Let $C = A + B$, find the value of C .

2. 有一批糖共 x 粒， x 为正整数，这批糖能分别为 851 人及 943 人所均分。求 x 的最小可能值。

A total number of x candies, x is a positive integer, can be evenly distributed to 851 people as well as 943 people. Find the least possible value of x .

3. 如图一，正四面体 $ABCD$ 的边长为 2 cm。若该正四面体的体积是 $\sqrt{R} \text{ cm}^3$ ，求 R 的值。

In Figure 1, $ABCD$ is a tetrahedron with side length 2 cm. If the volume of the tetrahedron is $\sqrt{R} \text{ cm}^3$, find the value of R .



图一

Figure 1

4. 已知 x 为正整数及 $x < 60$ 。若 x 恰有 10 个正因子，求 x 的值。

Given that x is a positive integer and $x < 60$. If x has exactly 10 positive factors, find the value of x .

5. 已知 $90^\circ < \theta < 180^\circ$ 及 $\sin \theta = \frac{\sqrt{3}}{2}$ 。若 $A = \cos(180^\circ - \theta)$ ，求 A 的值。

Given that $90^\circ < \theta < 180^\circ$ and $\sin \theta = \frac{\sqrt{3}}{2}$. If $A = \cos(180^\circ - \theta)$, find the value of A .

6. 设 x 为正实数，求 $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$ 的最小值。

Let x be a positive real number. Find the minimum value of $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$.

7. 设 x 及 y 为实数，且满足

$$\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5, \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5. \end{cases}$$

若 $z = x + y$ ，求 z 的值。

Let x and y be real numbers satisfying

$$\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5, \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5. \end{cases}$$

If $z = x + y$, find the value of z .

8. 设 R 为 $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ 被 4 除后的余数, 求 R 的值。

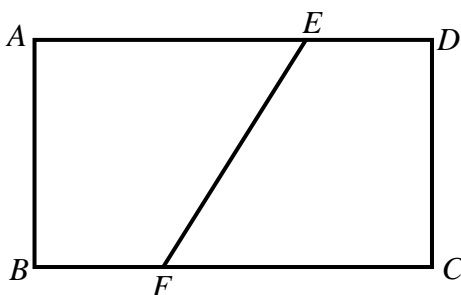
Let R be the remainder of $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ divided by 4. Find the value of R .

9. 已知 k 、 x_1 及 x_2 为正整数且 $x_1 < x_2$ 。设 A 、 B 及 C 为曲线 $y = kx^2$ 上的三点, 其 x 坐标分别为 $-x_1$ 、 x_1 及 x_2 。若 $\triangle ABC$ 的面积是 15 平方单位, 求所有可能 k 值的总和。

Given that k , x_1 and x_2 are positive integers with $x_1 < x_2$. Let A , B and C be three points on the curve $y = kx^2$, with x coordinates being $-x_1$, x_1 and x_2 respectively. If the area of $\triangle ABC$ is 15 square units, find the sum of all possible values of k .

10. 如图二, $ABCD$ 是长方形纸张并有 $AB = 4$ cm 及 $BC = 5$ cm。将该纸张对折, 使 C 点与 A 点重合, 得折痕 EF 。若 $EF = r$ cm, 求 r 的值。

In Figure 2, $ABCD$ is a rectangular piece of paper with $AB = 4$ cm and $BC = 5$ cm. Fold the paper by putting point C onto A to create a crease EF . If $EF = r$ cm, find the value of r .



图二

Figure 2